20. Calculate correct to six places of decimals.

$(1.01)^{3/2}-(0.99)^{3/2}$.

21. Sum the series to infinity :

$\frac{5}{1!}+\frac{7}{3!}+\frac{9}{5!}+\ldots\ldots$

22. Sum to n terms of the series

3.5.7+5.7.9+7.9.11+......

4186/M11 **MAY 2010**

Paper I — CALCULUS AND CLASSICAL ALGEBRA

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. If $y=\frac{1}{(x+1)(2x-1)}$, find y_n.

2. At which point is the tangent to the curve $x^2+y^2=5$ parallel to the line $2x-y+6=0$.

3. Find the radius of the curvature of the curve $x^4+y^4=2$ at the point $(1,1)$.

4. Show that $\int_0^{\pi/4} \log(1+\tan\theta)d\theta=\frac{\pi}{8}\log 2$.

5. Evaluate $\int(\log x)^2 dx$.

6. Expand $f(x)=x$ as a fourier series in the interval $(-\pi,\pi)$.

7. Show that any convergent sequence is a bounded sequence.

8. Discuss the convergence of the series

$$1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\ldots.$$

9. Show that the series $\Sigma\left|\frac{x^{n-1}}{(n-1)!}\right|$ converges absolutely for all values of n.

10. Sum to infinity, the series :

$$1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\ldots..$$

11. Show that :

$$\log\left(\frac{n+1}{n-1}\right)=\frac{2n}{n^2+1}+\frac{1}{3}\left(\frac{2n}{n^2+1}\right)^3+\frac{1}{5}\left(\frac{2n}{n^2+1}\right)^5+\ldots..$$

12. Sum the series :

$$\frac{3}{1^2.2^2}+\frac{5}{3^2.4^2}+\ldots.+\frac{2n+1}{n^2(n+1)^2}.$$

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Find the maxima (or) minimum values of $2(x^2-y^2)-x^4+y^4$.

14. For the curves $x^2=4y$ and $y^2=4x$, find the angle of intersection.

15. Show that the evolute of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ is another cycloid.

16. Show that $\int_0^\pi \theta\sin^3\theta d\theta=\frac{2\pi}{3}$.

17. If $f(x)=\begin{cases}-x & \text{in } -\pi<x<0 \\ x & \text{in } 0<x<\pi,\end{cases}$

expand $f(x)$ as a fourier series in $(-\pi,\pi)$.

18. State and prove Raabe's test.

19. Prove that in an absolutely convergent series, the series formed by its positive terms alone is convergent and the series formed by its negative terms alone is convergent and conversely.