18. Find the centre and radius of the circle $x^2 + y^2 + z^2 = 225$, $2x - 2y + z = 27$.

19. Find the equation of the right circular cylinder of radius 2 whose axis passes through $(1, 2, 3)$ and has direction cosines proportional to $(2, -3, 6)$.

20. (a) If the second order partial derivatives of $\vec{f}$ be continuous, then prove that, $div\left(curl\ \vec{f}\right) = 0$.

(b) Prove that

$div(\phi\overline{u}) = (grad\ \phi).\ \overline{u} + \phi\ div\ \overline{u}$.

21. Evaluate the surface integral $\iint\limits_S \left[yz\overline{i} + zx\overline{j} + xy\ \overline{k}\right] ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

22. Verify Green's theorem in the plane for $\vec{F} = \left(xy + y^2\right)\vec{i} + x^2\ \vec{j}$ over the closed curve bounded by $y = x$ and $y = x^2$.

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Paper II — TRIGONOMETRY, ANALYTICAL GEOMETRY OF 3 DIMENSIONS AND VECTOR CALCULUS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Calculate $(1 + i)^{10}$.

2. Prove that $\dfrac{\sin 6\theta}{\sin \theta} = 32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$.

3. If $\tan(x + iy) = u + iv$, prove that $\dfrac{u}{v} = \dfrac{\sin 2x}{\sinh 2y}$.

4. Find the equation of the plane through the point $(1, -2, 3)$ and the intersection of the planes $2x - y + 4z = 7$ and $x + 2y - 3z + 8 = 0$.

5. Find the equation of the plane which contains the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ on the plane $8x + 2y + 9z - 1 = 0$.

6. Show that the lines $\frac{x-4}{2} = \frac{y-5}{3} = \frac{z-6}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar and find the equation of the plane in which they lie.

7. Find the equation of the sphere which has its centre at $(6, -1, 2)$ and touches the plane $2x - y + 2z - 2 = 0$.

8. Find the volume of tetrahedron whose vertices are $(3, 2, 3)$, $(0, 3, 4)$, $(6, 1, 4)$, $(6, 3, 2)$.

9. Show that the equation of the right circular cone which its vertex at $(0, 0, 0)$ with z-axis as its axis and semi vertical angle equal to α is $x^2 + y^2 = z^2 \tan^2 \alpha$.

10. Find the directional derivative of $f(x, y, z) = z^2x + y^3$ at $(1, 1, 2)$, in the direction $\frac{1}{\sqrt{5}}\vec{i} + \frac{2}{\sqrt{5}}\vec{j}$.

11. Calculate the line integral of $(y^2 + z^2)\vec{i} + (z^2 + x^2)\vec{j} + (x^2 + y^2)\vec{k}$ over the straight line joining the points $(0, 0, 0)$, $(1, 1, 1)$.

12. Verify divergence theorem for $\vec{F} = \vec{r}$ and S is the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Separate with real and imaginary parts of $\tan^{-1}(\alpha + i\beta)$.

14. Sum the series upto n terms $\cos\alpha \cos 3\alpha + \cos 3\alpha \cos 5\alpha + \cos 5\alpha \cos 5\alpha + \ldots$.

15. (a) Solve $x^9 + x^5 - x^4 - 1 = 0$

(b) Find the value of i^i.

16. Find the equation of the plane which bisects the acute angle between the planes $3x - 4y + 12z = 26$ and $x + 2y - 2z = 9$.

17. Find the length and equations of the shortest distance between the lines

$$\frac{x-10}{1} = \frac{y-9}{3} = \frac{z+2}{-2};\ \frac{x+1}{2} = \frac{y-12}{4} = \frac{z-5}{1}.$$

21. (a) Solve $\frac{d^2y}{dx^2}+y=\operatorname{cosec}x$ by using method of variation of parameters.

(b) Eliminate h and K from $z=(x^2+h)(y^2+K)$.

22. Solve $y''-3y'+2y=e^{-t}$ given $y(0)=1$, $y'(0)=0$ using Laplace transforms.

Paper III — MODERN ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Define equivalence relation. Give an example.

2. If $f:A\to B$ and $g:B\to C$ are bijections then prove that $(g\circ f)^{-1}=f^{-1}\circ g^{-1}$.

3. Show that the union of two subgroups is a subgroup if and only if one is contained in the other.

4. Prove that every group of Prime order is cyclic.

5. If f is a homomorphism of a group G into a group G' with Kernel K then prove that K is a normal subgroup of G.

6. Prove that every finite integral domain is a field.

7. Solve : $(D^2-3D+2)y=\sin 3x$.