5531/KA1 **MAY 2011**

Paper I — ALGEBRA — I

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (4 × 10 = 40 marks)

Answer any FOUR questions.

1. (a) Prove that every permutation is a product of two-cycles.

   (b) Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

2. State and prove Cauchy's Theorem.

3. Define solvable group. If G is a group and N is a normal subgroup of Gm such that both N and G/N are solvable prove that G is solvable.

4. If R is commutative ring with unit element and M is an ideal of R prove that M is a maximal ideal of R if and only if R/M is a field.

5. Prove that $F[x]$ is an integral domain.

6. Prove that a polynomial of degree n over a field can have atmost n roots in any extension field.

7. (a) Let ch $f=0$ and $f(x)\in F[x]$ be a monic irreducible polynomial. Then prove that $f(x)$ is separable.

   (b) Let $P(x)\in K[x]$ be irreducible. Then prove that $P(x)$ is in separable if and only if $P'(x)=0$.

8. (a) Let F be a finite field. Then prove that F has p^m elements where the prime number p is the characteristic of F.

   (b) Prove that the multiplicative group of non zero elements of a finite field is cyclic.

SECTION B — ($3 \times 20 = 60$ marks)

Answer any THREE questions.

9. State and prove second part and third part of sylow's theorem.

10. (a) (i) State and prove Gauss Lemma

    (ii) State and prove Eisenstein criterion.

    (b) If G is finite and is the direct product of its sylow subgroups prove that G is nilpotent.

11. If R is a unique factorization domain prove that $R[x_1,x_2,....x_n]$ is also a unique factorization domain.

12. (a) If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F.

    (b) Prove that any finite extension of a field of characteristic o is a simple extension.

13. State and prove fundamental theorem of Galois Theory.

14. (a) (i) Prove that it is impossible, by straight edge and compass alone, to trisect 60^{o}.

    (ii) If F is a finite field and $\alpha\neq 0$, $\beta\neq 0$ are two elements of F, then prove that we can find elements a and b in F such that $1+\alpha a^2+\beta b^2=0$.

    (b) Let $q(x)$ be an irreducible polynomial of degree p, p a prime, over the field Q of rational numbers. Suppose that $q(x)$ has exactly two non real roots in the field of complex numbers, prove that the Galois group of $q(x)$ over Q is S_p, the symmetric group of degree p.

5532/KA2 **MAY 2011**

REAL AND COMPLEX ANALYSIS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

PART A — (4 × 10 = 40 marks)

Answer any FOUR questions.

1. Let $\{p_n\}$ be a sequence in a metric space X. Prove that

   (a) $\{p_n\}$ converges to $p \in X$ if and only if every neighbourhood of p contains all but finitely many points of the terms of $\{p_n\}$.

   (b) If $p_1 \in X$, $p_2 \in X$ and if $\{p_n\}$ converges to p_1 and p_2, then $p_1 = p_2$.

2. Prove that any continuous function on a compact metric space is uniformly continuous.

3. State and prove the generalised mean value theorem.