15. With the usual notation show that if $T \in A(V)$ has all its characteristic roots in F then there is a basis of V in which the matrix of T is triangular.

16. Prove with the usual notation :

(a) if $T \in A(V)$ is such that $\langle vt, v \rangle = 0$ for all $v \in V$, then $T = 0$.

(b) a linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

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M.Sc Maths (R) (11C, after)

2535/KM1 **OCTOBER 2011**

Paper I — ALGEBRA – I

(For those who joined in July 2009 and after)

Time : Three hours Maximum : 100 marks

PART A — ($4 \times 10 = 40$ marks)

Answer any FOUR questions.

1. When do you say that two elements of a group G are conjugates? Show that conjugacy is an equivalence relation. If $c(a)$ is the equivalence class containing a with C_α elements what can you say about ΣC_α?

2. For $s \in N$, G an abelian group, define $G(s) = \{x \in G : x^s = e\}$. Is $G(s)$ a subgroup of G? If G, G' are isomorphic finite abelian groups then show that for every integer s, $G(s)$, $G'(s)$ are isomorphic?

3. Let I be a two-sided ideal of a ring R. Show that R/I is a ring and is a homomorphic image of R.

4. Define the term Euclidean Ring. Show that if a, b are two elements of a Euclidean ring R then they have a greatest common divisor d and that $d = \alpha a + \beta b$ for some $\alpha, \beta \in R$.

5. Define the term primitive polynomial. Show that if $f(x)$, $g(x)$ are two primitive polynomial then their product is also a primitive polynomial.

6. Define an Inner Product Space. Establish the Schwarz inequality.

7. Define invertibility of $T \in A(V)$. Show that if V is a finite dimensional vector space over a field F, then $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial is non-zero.

8. Let V be a vector space over F. With the usual notation for $T \in A(V)$ define the Hermitian adjoint T^* of T. Show that

   (a) $(T^*)^* = T$.

   (b) $(S+T)^* = S^* + T^*$.

   (c) $(\lambda S)^* = \bar{\lambda} S^*$ and

   (d) $(ST)^* = T^* S^*$ for all $S, T \in A(V)$ and all $\lambda \in F$.

PART B — (4 × 15 = 60 marks)

Answer any FOUR questions.

9. Write the class equation. Let G be a finite group of order n and let p be a prime dividing n. Show that G has an element of order p. What is the number of conjugate classes in S_n?

10. State the Sylow's theorem and give the second proof.

11. Show that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.

12. State and prove

    (a) the division algorithm for polynomials.

    (b) the Eisenstein criterion.

13. Let V, W be finite dimensional vector spaces over a field F of dimension m, n respectively. Define $Hom(V, W)$. Show that $Hom(V, W)$ is a vector space of dimension mn over F.

14. Suppose A is an algebra with identity over a field F, show that with the usual notation, A is isomorphic to a subalgebra of $A(V)$ for some vector space V over F.

2536/KM2 **OCTOBER 2011**

ANALYSIS – I
(REAL AND COMPLEX ANALYSIS)

(For those who joined in July 2009 and after)

Time : Three hours Maximum : 100 marks

PART A — ($4 \times 10 = 40$ marks)

Answer any FOUR questions.

1. If $E_1, E_2, E_3, \cdots$ is a sequence of countable sets show that their union is again countable.

2. Suppose $K \subset X \subset Y$. Show that K is compact relative to X if and only if K is compact relative to Y.

3. State and prove the Root test or the Ratio test on the convergence of a series.

4. Let f be a continuous mapping of a compact metric space into another metric space. Prove that f is uniformly continuous.

5. State and prove the chain rule for differentiation.

6. If a, b are complex numbers show that $\left|\frac{a-b}{1-\bar{a}b}\right| = 1$ if and only if either $|a| = 1$ or $|b| = 1$.