4507/KA1 **MAY 2010**

Paper I — ALGEBRA — I

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

PART A — (4 × 10 = 40 marks)

Answer any FOUR questions.

Each question carries 10 marks.

1. State and prove Lagrange's theorem.

2. State and prove Cauchy's theorem for Abelian groups.

3. Prove that the homomorphic image of a solvable group is solvable.

4. Define maximal ideal. Let R be a commutative ring with unity. Prove that an ideal M of R is maximal ideal of R if and only if R/M is a field.

5. Let $\alpha \in R$ be algebraic over F and $f(x)$ the minimum polynomial of α with deg $f(x)=n$. Then prove that $F(\alpha)$ has dimension n as a vector space over F.

6. Prove that two elements α and α' are conjugate over F if and only if they have the same minimum polynomial over F.

7. Let F be a field of characteristic O. Show that every algebraic extension of F is separable.

8. Let K/F be a Galois extension of degree n, $(n, \rho)=1$ such that $G=G(K/F)$ is a solvable group. Then there exists a radical extension L/F such that $K \subset L$.

PART B — (3 × 20 = 60 marks)

Answer any THREE questions.

Each question carries 20 marks.

9. Define P-sylow subgroup of a group. If ρ is prime number and $\rho^{\alpha}/O(G)$ then prove that G has a subgroup of order p^{α}.

10. (a) Let R be an Euclidean ring. Suppose that for $a, b, c \in R$, a/bc but $(a, b)=1$ then prove that a/c.

(b) Prove that the field C of all complex numbers is algebraically closed.

11. (a) Prove that if R is a commutative ring with unit elements, so is $R[x]$. If R is an integral domain then prove that $R[x]$ is also an integral domain.

(b) If f is a field then prove that $f(x)$ is an Euclidean domain.

12. Let K be an extension field of F. Prove that the element $a \in K$ is algebraic over F if and only if $F(\alpha)$ is a finite extension of F.

13. State and prove Artin's theorem.

14. (a) Let G be a finite abelian group enjoying the property that the relation $x^n = e$ is satisfied by atmost n elements of G is a cyclic group.

(b) Prove that it is impossible by straight edge and compass alone to trisect 60°.

13. Show that a nonempty open set in the plane is connected $\Leftrightarrow$ any two of its points can be joined by a polygon which lies in the set.

14. State and prove the Cauchy theorem for rectangle.

4508/KA2 MAY 2010

Paper II — REAL AND COMPLEX ANALYSIS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

PART A — $(4 \times 10 = 40$ marks)

Answer any FOUR questions.

1. (a) Define a continuous function on a metric space.

   (b) Show that f is a continuous function on a metric space $\Leftrightarrow$ Inverse image of an open set in y (under f) is open in X.

2. State and prove the Taylor's theorem for real functions on $[a,b]$.

3. (a) If f is continuous on $[a,b]$ show that $f \in \boldsymbol{R}(\alpha)$ on $[a,b]$.

   (b) If f is monotonic on $[a,b]$ and α is continuous on $[a,b]$ then show that $f \in \boldsymbol{R}(\alpha)$.