7492/KA1 **OCTOBER 2008**

Paper I — ALGEBRA — I

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

PART A — $(4 \times 10 = 40$ marks)

Answer any FOUR questions.

1. If G is a finite group, prove that the number of elements conjugate to a in G is the index of the normalizer of a in G.

2. If $O(G) = p^2$ where p is a prime number then prove that G is abelian.

3. Prove that a finite integral domain is a field.

4. Let R be a commutative ring with unit element whose only ideals are (O) and R itself. Then prove that R is a field.

5. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

6. State and prove Gauss' lemma.

7. If $f(x)$ and $g(x)$ are primitive polynomials then prove that $f(x)g(x)$ is a primitive polynomial.

8. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

PART B — $(3 \times 20 = 60$ marks)

Answer any THREE questions.

9. State and prove Sylow theorem (first part).

10. Prove that $J[i]$ is a Euclidean ring.

11. Prove that every integral domain can be imbedded in a field.

12. If L is a finite extension of K and if K is a finite extension of F then prove that L is a finite extension of F.

13. If F is of characteristic 0 and if a, b are algebraic over F, then prove that there exists an element $C \in F(a, b)$ such that $F(a, b) = F(C)$.

14. If p is a prime number and $p / O(G)$ then prove that G has an element of order p.

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7493/KA2 **OCTOBER 2008**

Paper II — REAL AND COMPLEX ANALYSIS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

PART A — ($4 \times 10 = 40$ marks)

Answer any FOUR questions.

1. Prove that union of countable set is countable.

2. Suppose $\{s_n\}$ is monotonic. Then prove that $\{s_n\}$ converges if and only if it is bounded.

3. Prove that a mapping f of a metric space X into a metric space Y is continuous on X of and only if $f^{-1}(V)$ is open in X for every open set V in Y.

4. State and prove mean value theorem.

5. State and prove the fundamental theorem of calculus.

6. If a_i, b_i $(i = l \text{ to } n)$ are complex numbers prove that $\left|\sum_{i=1}^{n} a_i\, b_i\right|^2 \leq \sum_{i=1}^{n} |a_i|^2 \sum_{i=1}^{n} |b_i|^2$.

7. The cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

8. If $f(z)$ is analytic and non-constant in a region Ω, then prove that its absolute value $|f(z)|$ has no maximum in Ω.

PART B — (3 × 20 = 60 marks)

Answer any THREE questions.

9. Prove that $\lim_{n-\alpha} \left(1+\frac{1}{n}\right)^n = e$.

10. Let f be a continuous mapping of a compact metric space X into a compact metric space Y. Then prove that f is uniformly continuous on X.

11. State and prove Taylor's theorem.

12. Assume that α increases monotonically and $\alpha' \in R$ on $[a, b]$ let f be a founded real function on $[a, b]$. Then prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R$. In that case prove also that $\int_a^b f d\alpha = \int_a^b f(x)\, \alpha'(x)\, dx$.

13. State and prove weierstrass theorem on Analytic function.

14. Prove that a set is compact if and only if it is complete and totally bounded.

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