5. (a) (i) Find the solution of Poisson's equation $\nabla^2 \phi = -\rho(r)$ using Green's function. (12)

(ii) Find the Green's function for the boundary value problem $\frac{d^2 y}{dx^2} - k^2 y = f(x)$ with boundary conditions $y(\pm\infty) = 0$. (8)

Or

(b) Explain the stretched string wave equation. (20)

4529/MP1 **MAY 2010**

Paper I — MATHEMATICAL PHYSICS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

All questions carry equal marks.

1. (a) (i) State and prove Green's theorem in vector analysis. (10)

(ii) Explain outer product and inner product of tensors. (10)

Or

(b) (i) State and prove Cayley-Hamilton theorem. (10)

(ii) Obtain an expressions for grad ϕ and div A in spherical polar co-ordinates. (10)

2. (a) (i) Find the Fourier Series for the periodic function $f(x)$ defined by

$$f(x) = -\pi \text{ if } -\pi \le x \le 0$$
$$= x \quad \text{if } 0 \le x \le \pi$$

Hence prove that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \qquad (12)$$

(ii) Prove that $\beta(m, n) = \frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}$. (8)

Or

(b) (i) Prove that $\lim_{n \to \infty} \int_{-\infty}^{\infty} \phi_n(x) f(x)\, dx = f(0)$ where $\phi_n(x) (n = 1, 2, 3 \cdots)$ as a delta sequence. (12)

(ii) Prove that $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$ where $f(x)$ is a continuous function. (8)

3. (a) (i) State and prove Cauchy's Residue theorem. (10)

(ii) Prove that $u(x, y) = x^2 - y^2$ is a harmonic function. Find $v(x, y)$. So that $f(z) = u + iv$. (10)

Or

(b) (i) Show that Fourier transform of a Gaussian probability function is also a gaussian probability function. (10)

(ii) Find the Fourier sine and cosine transforms for the function $f(x) = 2x \;\; 0 < x < 4$. (10)

4. (a) (i) Prove that $P_n(x) = 2^{\frac{1}{n}} n! \frac{d^n}{dx^n} (x^2 - 1)^n$. (10)

(ii) Prove that $x J_n^1(x) = n J_n(x) - x J_{n+1}(x)$. (10)

Or

(b) (i) Solve the Laguerre's differential equation using power series technique. (10)

(ii) Prove that

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$

$$J_{\frac{3}{x}}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{\sin x}{x} - \cos x \right]. \qquad (10)$$

4530/MP2 **MAY 2010**

CLASSICAL MECHANICS AND STATISTICAL MECHANICS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

All questions carry equal marks.

1. (a) (i) State and derive D'Alembert's principle. (8)

(ii) Obtain Lagrange's equation of motion for a holonomic conservative system using D'Alembert's principle. (12)

Or

(b) (i) State and prove conservation of linear momentum. (10)

(ii) Define generalized coordinate and obtain the expression for generalized momentum. (10)

2. (a) (i) State and prove Bertrand's theorem. (12)

(ii) Derive the motion in time in the Kepler problem. (8)

Or

(b) Deduce equations of motion and first integrals. (20)

3. (a) (i) Derive the Euler's equation of motion for a rigid body. (8)

(ii) Explain normal coordinates. (12)

Or

(b) (i) State and prove the principle of least action. (12)

(ii) Derive the Hamilton's equations from a variational principle. (8)

4. (a) (i) Describe the harmonic oscillator problem as an example of canonical transformation. (15)

(ii) Deduce equation of motion interms of Poisson's bracket. (5)

Or

(b) Solve Kepler problem applying action angle variable. (20)

5. (a) (i) Distinguish between Fermions and Bosons. (5)

(ii) Obtain the Maxwell-Boltzmann distribution function. (15)

Or

(b) Explain the thermodynamic properties of a system. (20)
