Paper I — MATHEMATICAL PHYSICS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

All questions carry equal marks.

(5 × 20 = 100)

1. (a) (i) State and prove Stokes theorem.

(ii) Verify Stokes theorem for the vector $F = (z, x, y)$ taken over the half of the sphere $x^2 + y^2 + z^2 = a^2$ lying above the xy plane.

Or

(b) Define contravariant and co-variant tensors? And show that the law of transformation for a contravariant tensors is transitive.

2. (a) (i) Deduce the value of $\Gamma(1/2)$.

(ii) Find the relation between Beta and Gamma function.

(iii) Show that $\beta(m, n) = \beta(n, m)$.

Or

(b) Distinguish Dirac delta function from kronecker delta function and show that $\delta(x^2 - a^2) = 1/2a\,\{\delta(x + a) + \delta(x - a)\}$ where $a > 0$.

3. (a) Derive the Cauchy-Riemann's equation
$f(z)$ is expressed in polar coordinates.

Or

(b) Find the cosine transformation of $X^n e^{-ax}$.

4. (a) (i) Starting from the definition of $J_n($
prove that $J_{n-1}(x) + J_{n+1}(x) = 2n/x\, J_n(x)$.

(ii) Obtain the series solution of th
Hermite differential equation $y'' - 2xy' + 2ny = 0$ whe
$n = 2$.

Or

(b) (i) Prove that following recurren
relation for the Laguerre polynomi
$L_n(x) - nL'_{n-1} + nL_{n-1}(x) = 0$.

(ii) Construct the polynomial solution f
Legenders differential equation for $m = 0$.

5. (a) Derive the wave equation for a perfe
flexible stretched string and then construct a Fouri
series solution for it.

Or

(b) Construct the Greens function for the no
homogeneous problem $d^2u/dx^2 = f(x)$ with th
boundary conditions $u(0) = u(1) = 0$.

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Paper II — CLASSICAL MECHANICS AND STATISTICAL MECHANICS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

All questions carry equal marks.

1. (a) Obtain Lagrange's equations of motion using D'Alembert's principle for holonomic system. Show that if the Lagrangian does not depend explicitly on time, then the energy is conserved.

Or

(b) State Hamilton's principle and derive Lagrange's equations of motion from it. How will the result be modified if the forces are non-conservative?

2. (a) Define Euler's angles and derive the Euler's equations of motion in terms of Euler's angles.

Or

(b) What is meant by the "laboratory system" and the "centre of mass system" in a two body scattering problem? How will you transform the differential cross-section, energy and scattering angle from the centre of mass system to the laboratory system?

3. (a) (i) Describe the Torque free motion of a rigid body.

(ii) Derive the relationship between the angular momentum and the angular frequency.

Or

(b) Outline the theory of small oscillations of a system about an equilibrium position and apply it to the oscillations of a symmetric linear triatomic molecule.

4. (a) Explain the Hamilton-Jacobi method. Illustrate it by solving the problem of a simple harmonic oscillator.

Or

(b) What are action and angle variables? Discuss how they are applied to the Kepler's problem.

5. (a) Derive the Bose-Einstein distribution law. Use it to deduce the Planck's law for black body radiation.

Or

(b) Show that at low temperatures the specific heat C_v of an ideal Fermi gas varies linearly with temperature and has the limiting value zero at $T = 0$.