

MATHEMATICAL PHYSICS

(For those who joined in July 2003 and after)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

All questions carry equal marks.

1. (a) (i) State and prove Stoke's theorem in space.
- (ii) Show that $\iiint_S \vec{r} \cdot \hat{n} ds = 3V$, where V is the volume enclosed by S and \vec{r} is the position vector.

Or

- (b) (i) State and prove Cayley-Hamilton theorem.
- (ii) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ 3 & 1 & 2 \end{bmatrix}.$$

2. (a) (i) Obtain the Fourier series of $f(x) = x$ in the interval $-\pi \leq x \leq \pi$.
- (ii) Show that $p\beta\{p, (q+1)\} = q\beta\{(p+1), q\}$.

Or

- (b) (i) Distinguish between Dirac delta function from Kronecker delta function.
- (ii) Show that $\lim_{n \rightarrow \infty} \int_{-\infty}^{+\infty} \phi_n(x) f(x) dx = f(0)$ where $\phi_n(x)$ ($n = 1, 2, 3$) as a delta sequence.
3. (a) (i) State and prove Cauchy's Residue theorem.

(ii) Evaluate $I = \int_0^{2\pi} \frac{d\theta}{2 + \cos \theta}$.

Or

- (b) (i) State and prove the convolution theorem for Fourier Transform.
- (ii) Find the Fourier sine and cosine transforms for the functions $f(x) = 2x$ $0 < x < 4$.

4. (a) (i) Write Sturm-Liouville's equation and deduce Legendre, Bessel and Hermite differential equations from it.
- (ii) Construct a polynomial solution of Bessel differential equation.

Or

- (b) Show that $P_n(x)$ is the coefficient of z^n in the expansion of $(1 - 2xz + z^2)^{-\frac{1}{2}}$ in ascending powers of z .
5. (a) (i) Derive Green's function for Poisson's equation.
- (ii) Derive the Green's function for one dimensional problem.

Or

- (b) Derive the wave equation for a flexible stretched string and then construct a Fourier series solution for it.

Paper II — CLASSICAL MECHANICS AND
STATISTICAL MECHANICS

(For those who joined in July 2003 and after)

Time : Three hours

Maximum : 100 marks

Answer ALL questions

1. (a) Using D' Alembati's principle obtain Lagrange's equations of motion for a conservative system.

Or

 (b) (i) What is a cyclic co-ordinate? (2)
 (ii) Show that the generalised momentum conjugate to a cyclic co-ordinate is conserved. (8)
 (iii) Discuss the advantages of variational principle. (10)
2. (a) (i) Using Laplace-Runge- lenz vector derive the orbit equation for the kepler problem. (8)
 (ii) Explain Inverse square law of force in kepler's problem. (12)

Or

 (b) Discuss
 - (i) Lagrange's equations of motion and first integral. (12)
 - (ii) Equivalent one- dimensional problem. (8)

3. (a) (i) Calculate angular velocity of precession and spin where mutation is absent
 (ii) What are normal co-ordinates?

Or

- (b) State and prove the principle of least action.

4. (a) Derive Equations of Motion and infinitesimal canonical transformation in the poisson Bracket formulation.

Or

- (b) Discuss the symplectic approach to canonical transformation.

5. (a) (i) State and prove the principle of equipartition of energy.
 (ii) Discuss the quantized linear oscillation.

Or

- (b) Explain the specific heat capacity of a diatomic gas. Also explain Fermi-Dirac statistics.

